A fundamental question about the speed of electromagnetic and gravity waves

Gregory Loew, Professor Emeritus

SLAC National Accelerator Laboratory

The question I am examining in this article is whether the value of the speed of electromagnetic and gravitational waves in free space is a fundamental constant of nature or whether it can be derived from theory.

Experimentally, in the current (2019) "*Système International d'Unités*" (SI) , the speed of light in vacuum, commonly denoted by the letter **c** (from "celeritas" or speed in Latin), is set exactly at **299 792 458 meters per second**. In this system the meter is derived from this value by the distance traveled by light in one second, and the second is defined as the length of time occupied by **9,192,631,770 cycles** of the radiation emitted by a cesium-133 atom in a transition between two specified hyperfine energy states. In practical terms this means that light travels one meter in approximately 3.3 nanoseconds (one nanosecond per foot), and that it takes about 500 seconds to travel the 150,000,000 kms between the sun and the earth.

Note that with this definition, if it were to be measured that the speed of light is slightly different, the meter would be changed accordingly, and the speed of light would still be given by the defied value.

Properties of c

As pointed out by my colleague, Prof. Fred Gilman at Carnegie Mellon, the fact that the speed of light **c** appears in all the physical phenomena enumerated below is the direct result of Einstein's Theory of Special Relativity (SR) which states that the speed of light in vacuum is the same in any inertial reference frame. This fact had been confirmed by the Michelson Morley experiment which eliminated the possibility of an ether as it was then conceived.

1) In classical Maxwellian physics, **c** is the phase and group velocity of a plane electromagnetic wave of any frequency in vacuum and free space.
2) The electromagnetic wave is a signal that is generated when an electric charge is accelerated/decelerated at a point, as in an atom, an antenna, a synchrotron, a laser, etc. It travels as a dipole mode.
3) In modern electrodynamics, it is the speed of a photon[1] (quantum of light) or train of photons in vacuum, over a distance greater than the width of an atom[2]. These photons have no mass, an energy hν where h is Planck's constant and ν is their frequency, and a momentum hν/c. Unlike the plane waves, the photons from a monochromatic source have a phase at the instant of emission but this phase does not change as they move through the vacuum.
4) It is the presumed speed of the gluons inside nucleons (although it probably cannot be measured).
5) In cosmology, it is the speed of propagation of gravitational waves in free space. From recent observations of neutron star mergers, it is equal to the speed of light within a fractional accuracy of less than 10^{-15}. In other words, $\mathbf{c}_{em}=\mathbf{c}_{gr}$. It travels as a quadrupole mode.

---

[1] It is ironic that about a year before his death in 1955, Albert Einstein lamented in a letter to his friend Michele Besso that although he had thought exhaustively about the photon (the quantum of light) for fifty years, he still couldn't understand "what it is."

[2] Richard P. Feynman, "QED," page 90, Fig.56, Princeton University Press, 1985. It's not clear to me what is special about this distance.

6) If gravitons or quanta of gravitational energy are ever observed, it is indeed guessed that their speed will also be **c**. For this to happen, a particle with spin 2 will have to be found.
7) It is the maximum asymptotic speed at which a massive particle can travel in vacuum.
8) For the above reason, it was why for the first three years of my career when I was designing the SLAC three-km accelerator structure, I had to make sure it had a constant phase velocity **c** at the operating frequency of the machine (2856 MHz), essentially synchronous with the relativistic electrons.
9) It is the maximum speed at which information can be transmitted (ignoring "quantum entanglements" which can result in instantaneous communication but not in information transfer).
10) Experts generally believe that the speed of light has probably not changed since the time of recombination.

Lorentz invariance

The speed of propagation of a plane electromagnetic wave or a photon in free space and vacuum is independent of its frequency, its direction of propagation in space, its polarization, the speed of its source, and the inertial frame of reference of the observer. This last point is equivalent to saying that **Maxwell's wave equation is Lorentz invariant.** If we used Galilean coordinate transformations from one inertial reference frame to another for the wave equation, its form would change, but if we use Lorentz transformations, it remains the same: the equation is **unchanged.** Note that the formulas for the Lorentz transformations for space and time coordinates both contain the term $\mathbf{(v/c)^2}$ where v is the velocity between the two inertial frames. However, nowhere in these formulas is the value of c = 299,792,458 m/s to be found in these units or any other units. For all we know, it could be twice or half this value, and Lorentz invariance would still hold. This is the puzzle I am addressing in this article: Where does this number come from?

Furthermore, if the source of light moves away from the observer, it is seen that the frequency of the wave is red shifted but its speed remains constant. Similarly, if the source moves towards the observer, the light is blue shifted but again, its speed remains the same. In neither case is energy conservation violated because the energy is not conserved from one reference frame to the other (the receding star is one frame, the observer on earth is another).

Fundamental parameters of nature and units

Is the speed of light a fundamental parameter of nature? On this question it is worthwhile referring to two (out of many) articles. One is by John Baez [3] and the other is a trialogue between Lev Okun, Gabriele Veneziano and Michael Duff [4]. Baez counts that there are 26 **dimensionless** fundamental parameters, but he discounts any of the

---

[3] John Baez, "How many fundamental constants are there?" April 22, 2011, baez@math.removethis.ucr.andthis.edu

[4] *Duff, M. J.; Okun, L. B.; Veneziano, G. (2002). "Trialogue on the number of fundamental constants". Journal of High Energy Physics. 2002 (3): 023. arXiv:physics/0110060. Bibcode:2002JHEP...03..023D. doi:10.1088/1126-6708/2002/03/02.*

parameters with dimensional units. He has a skeptical view about these: "…in the grand scheme of things, units are not very important. They are arbitrary human conventions. As long as you stick with *some* choice or other you will do okay". Okun argues that there are three fundamental parameters with **dimensionful** units, the speed of light c, the quantum of angular momentum and of action $\hbar$ (h divided by 2π) and Newton's gravitational constant G. Veneziano argues that there are only two such units, c and the length of a string $l_{s.}$ Finally, Duff believes that there are no fundamental parameters with dimensional units.

These beliefs are consequential because if a parameter with dimensions is fundamental, presumably it cannot be calculated. It is almost like a metaphysical belief. However, as Okun[5] says, "we defined as fundamental those constants which cannot be calculated at our present level of fundamental knowledge (or rather ignorance)." We have no model for the properties of the photon and the properties of the vacuum (if any) to make such a calculation at this time.

Note that Okun's position was consistent with Planck's in that it was on the basis of c, $\hbar$ and G that Planck derived the units of Planck length, time and mass. The Planck length of 1.055×10^{-34} m is close to the length of a string. In these very small units, light travels at the speed of 1 Planck length per Planck time [Dimensions LT^{-1}]. $\hbar$ is also 1 {Dimensions L^2MT^{-1}] and G is 1 [Dimensions $L^3M^{-1}T^{-1}$.

[5] Lev Okun, a remarkable physicist, coined the name "hadrons" for strongly interacting particles, and pointed out that the famous Einstein formula $E=mc^2$ should really be written as $E_0=mc^2$..

Hints from classical electrodynamics

In classical electrodynamics, according to Maxwell's equations and the wave equation in the SI MKS system, the velocity c of a monochromatic plane electromagnetic wave of any frequency in vacuum along a straight direction obeys the following relation:

$$\mathbf{c = 1/ (\varepsilon_0.\mu_0)^{1/2}} \qquad \textbf{(1)}$$

where $\boldsymbol{\varepsilon_0}$ and μ_0 are respectively the permittivity and permeability of free space:

ε_0 = 8.854 187 817... x 10^{-12} F/m (farads per meter) and $\mu_0 \sim 4\pi\times10^{-7}$ H/m (henrys per meter). In the 2019 SI system of units mentioned earlier, both these constants are now measured. The measurements do not give us an insight into where they come from but tell us that somehow, the vacuum has these properties if we choose these units.

We can get a physical feeling for the above formula. The same velocity of propagation is observed for plane waves in coaxial lines, along two-wire transmission lines and slab-lines in vacuum. In analogy with the wave delay in a series/parallel LC network, these transmission systems transmit electric currents that must successively flow through their equivalent series inductance per unit length and charge the equivalent parallel capacitance per unit length, which in turn discharges itself through the next inductance, and so on.

If the medium changes from vacuum to some partially transparent dielectric like air, water or glass, the speed **v** is reduced as a function of the index of refraction of the medium **n** as

$$\mathbf{v = c/n} \quad \text{where} \quad \mathbf{n = (\varepsilon_r . \mu_r)^{1/2}} \qquad \textbf{(2)}$$

and ε_r and μ_r are respectively the relative permittivity and permeability of the medium that yield the net increased values $\varepsilon_r\varepsilon_0$ and $\mu_r\mu_0$. Note the multiplication sign.

Can a theoretical explanation be found with 20th Century Physics?

Based on what has been established so far in this article, we could stop here and say that in our universe, massless energy travels through space-time at c = 299,792,458 m/s, a fundamental constant of nature, and leave it at that.

However, for the last few years that I have been wondering about this question, I have asked myself if a further explanation can be found in theories like General Relativity (GR), Quantum Mechanics (QM), Quantum Electrodynamics (QED) or Quantum Field Theory (QFT)? What about Vacuum Fluctuations (VF) and the Casimir effect? As it turns out, the explanation must meet stringent requirements: it must be independent of the frequency of the quantum of energy, it cannot involve any losses, it must be Lorentz invariant, isotropic in all directions, and it cannot be dependent on the intensity of the electromagnetic or gravitational radiation.

Apparently, people like Oppenheimer, Furry, Pauli, Weisskopf, Dicke and others (see refs.1-4 in Mainland and Mulligan article below) had been thinking about this problem for quite some time. Just about one year ago, Mainland and Mulligan published an article[6] based on the idea that vacuum fluctuations are polarized by incident photons, an effect that results in a calculable value of $\boldsymbol{\varepsilon_0}$ and therefore of **c.** Their model is a *tour de force* that uses QM, QED, QFT but not the Casimir effect.

Specifically, the model treats the vacuum as a dielectric filled with electron-positron vacuum fluctuations that appear in the vacuum as transient parapositronium atoms that have spin zero as well as vacuum fluctuations of muon-antimuons and tau-antitaus, neglecting the minimal effects of quark-antiquark pairs. Incident photons polarize these pairs, which are at rest in the vacuum, similarly to the way they would polarize a real molecular dielectric.

The value of the speed of light c in the vacuum calculated by Mainland and Mulligan satisfies the two postulates of special relativity:

1) The laws of physics are the same in all inertial reference frames.
2) The speed of light in the vacuum has the same value c in every direction in all inertial reference frames.

Mainland and Mulligan calculate the speed of light in the vacuum using Maxwell's equations and properties of the vacuum. According to the above first postulate, these equations are the same in every inertial reference frame. Also, since the vacuum is the same in any reference frame and is homogeneous, the calculated value of c is the same in every direction in all inertial reference frames. Thus, their calculation satisfies the above second postulate of special relativity, eliminating the need for the postulate.

---

[6] Mainland, G.B., Mulligan, B. Polarization of Vacuum Fluctuations: Source of the Vacuum Permittivity and Speed of Light. *Found Phys* **50,** 457–480 (2020). https://doi.org/10.1007/s10701-020-00339-3

From the definition of $\boldsymbol{\varepsilon_0}=(6\boldsymbol{\mu_0}/\pi)\ (8e^2/\hbar)^2$, the model calculates the value of $\boldsymbol{\varepsilon_0}$ as $9.10 \times 10{-}12$ Farads/meter which is 2.7% higher than the accepted value. The speed of light **c** is then derived by plugging this value of $\boldsymbol{\varepsilon_0}$ into formula (1) in the previous paragraph. The vacuum permeability used here is $\boldsymbol{\mu_0}=2\alpha h$ where α, the fine structure constant, is calculated to be 1/138.9 The calculated speed of light yields 296000 km/s, or 1.4% lower than measured. The discrepancy would be even smaller if the value of $\boldsymbol{\varepsilon_0}$ were not just calculated to lowest order in α.

In the conclusion the article points out that in cosmology, before the time of recombination, the number of fermion-antifermion vacuum fluctuations was probably lower and the speed of light might have been higher.

Altogether, the Mainland and Mulligan article makes several assumptions that only expert theorists can validate. But if they are correct, it is a remarkable piece of work. Note that it talks only about electromagnetic waves and does not touch upon gravitational waves. Presumably, GR and the observations from simultaneous gravitational and electromagnetic waves from neutron stars confirm that $\mathbf{c}_{em}=\mathbf{c}_{gr}$ but it may be that this was not true during inflation and before recombination. This could have interesting consequences that may be observable in the future.

Before I discovered the above Mainland and Mulligan article, I thought that maybe the effect of VF that cause the Casimir effect could allow us to detect a variation in the value of $\boldsymbol{\varepsilon_0}$ as one decreases the spacing d (down to sub-micron scales) between the plates with area A of a vacuum condenser with capacity $C = \boldsymbol{\varepsilon_0} A/d$. This would show that $\boldsymbol{\varepsilon_0}$ is a function of the environment and the boundary conditions. Similarly, the speed of propagation of an electromagnetic wave in a slab line with sub-micron spacing of the conductors might be higher than c. However, given the comments in the article about the Casimir effect, I realize that this is a complicated problem, and I am shelving the idea of attempting to make these measurements.

In conclusion, unless demonstrated otherwise, the Mainland and Mulligan article seems to be the definitive answer to my question.

Acknowledgements

The author thanks Gordon Bruce Mainland of Ohio State University for his excellent suggestions to improve this article, and acknowledges useful discussions on this subject with his SLAC colleague Michael Peskin, and with Fred Gilman at Carnegie Mellon.